MORTALITY INVESTIGATION IN THE GOVERNMENT SERVICE INSURANCE SYSTEM

by

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Since the Government Service Insurance System started operation in 1937, no attempt has ever been made to investigate the mortality experience of the members of the System. It was not considered necessary to revise the old Hunter's Mortality Table which was used in the calculation of the premiums and the reserves of life insurance policies issued by the System, inasmuch as these policies were participating so that gains in mortality were eventually returned to the policyholders in the form of annual dividends. But in connection with the valuation of the Retirement Fund which was established in 1951, it is necessary to have a mortality table which will reflect more or less the mortality rates of the members of the System. As a matter of fact, it should be noted that for a number of years actuaries have observed decreases in annuity mortality rates and more recently have given thought to forecasting future decreases in annuity mortality. From this point of view an annuity mortality table should not only be conservative in regard to current experience but should make some allowance for future decreases.

It may be stated here that the purpose of the investigation of the experience of the members of the System is to determine not only the rate of mortality but also the rates of disability, of withdrawal and of age retirement for active employees of the System. With these four decrement rates in mind, it appears conservative to confine the investigation to employees who have at least five years of service. Rates based in such data will then be ultimate rates which will be conservative for withdrawals although not conservative for mortality of active employees. For the valuation of past service benefits of the Re-

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tirement Fund these rates would be more appropriate than rates based on all years of service.

In order to reflect as much as possible the experience of the Retirement Fund of the System, a three year observation period from July 1, 1951 to July 1, 1954 was used in the investigation. A census method of determining exposures for active employees was considered adequate. Separate mortality rates were constructed for the male and the female employees as it was observed that the mortality rates for female employees at all ages were lower than the corresponding rates of mortality for male employees.

In tabulating the exposures and the number of deaths of active employees as of July 1 of each year, the following notations and formulas were used:

P_x^{51} = number of active male employees as of July 1, 1951 who were born in year (1951 − x) and who were employed in 1946 or earlier.

P_x^{52} = number of active male employees as of July 1, 1952 who were born in year (1952 − x) and who were employed in 1947 or earlier.

P_x^{53} = number of active male employees as of July 1, 1953 who were born in year (1953 − x) and who were employed in 1948 or earlier.

$d_x^{51/52}$ = number of male employees who were in service as of July 1, 1951, were born in the year (1951 −x), were employed in 1946 or earlier and died before July 1, 1952; that is, those of the P_x^{51} employees as of July 1, 1951 who died before July 1, 1952.

$d_x^{52/53}$ = number of the P_x^{52} employees as of July 1, 1952 who died before July 1, 1953.

$d_x^{53/54}$ = number of the P_x^{53} employees as of July 1, 1953 who died before July 1, 1954.

E_x = $P_x^{51} + P_x^{52} + P_x^{53}$ = exposure between ages x and x +1 in the period July 1, 1951 to July 1, 1954.

d_x = $d_x^{51/52} + d_x^{52/53} + d_x^{53/54}$

Comparing the actual deaths, with the expected deaths using first Hunter's Tropical Mortality Table and then the Commissioners' 1941 Standard Ordinary Mortality Table, it was found that the Commissioners' Standard Ordinary Table gives nearer expected values to the actual deaths. So it was decided to use the Commissioners' Standard Ordinary Mortality Table in the graduation of the rates of mortality by the graphic method. Table I gives the comparison of the actual to the expected deaths using the Commissioners' Standard Ordinary Table.

In the graphic method of graduation the exposures and deaths at different ages were first grouped in order to reduce the irregularities especially at the younger and older ages. Table II gives the grouping of exposures. In calculating the average age for each group, each age is weighted in proportion to the expected deaths. It may be mentioned that a preliminary grouping was first made before the grouping shown in Table II was finally arrived at. In the preliminary grouping method, the rough rates of mortality without grouping were first plotted and then a smooth curve representing the general run of the values was sketched in lightly. From this curve the groups were chosen in such a way that points above this guide curve were balanced by points below it. The aim of the grouping was to insure that the group rate would lie close to the general run of the values.

The ratios of the actual deaths to expected deaths tabulated in the last column of Table II were plotted in a graphing paper (see graph at end of paper). The dotted line joining these points represents the ungraduated curve. The smooth graduated curve was drawn in such a way that it passed fairly close to the points representing the group rates. The percentage corresponding to each age was then read off and the graduated value of q_x computed by taking this percentage of the corresponding value of q_x according to the Commissioners' Standard Ordinary Table. The new expected deaths were then computed as shown in Table III.

The graduated mortality rates q_x tabulated in Table III were tested for smoothness and closeness of fit or adherence to data. The test for smoothness consisted in finding the average size of the third order differences of q_x. Table IV gives the first, second and third order differences. The average absolute value of third order differences was found to be .00005 which shows that the graduation was quite smooth.

In testing the closeness of fit or adherence to data, the Chi square test was used. The value of Chi square for each age was computed from the formula:

$$\text{Chi square} = \frac{(\text{Actual deaths} - \text{Expected deaths})^2}{E_x \, q_x \, p_x}$$

where $E_x \, q_x \, p_x$ is equal to the square of the standard deviation.

Table V shows the tabulation of the Chi square for the different ages. Since the number of degrees of freedom is more than 40, the Chi square distribution approximates a normal curve. From the table of areas of the normal curve we find that the probability of obtaining a value of Chi square as great as 63.928 or greater is about 34% which value is greater than a minimum 5% level.

It may be stated that considerable difficulty was encountered in connection with the Chi square test. While the first few attempts in drawing the graduated curves easily passed the smoothness test, when tested for closeness of fit, all of them failed to pass the test. Numerous corrections and hand-polishing had to be made before the graduated rates shown in Table III finally passed both the smoothness and closeness of fit tests. The same experience and the same trouble were also encountered in connection with the graduation of the rates of mortality for female employees.

The graduation of the mortality rates at the younger and older ages presented considerable difficulty due to the very small exposures at these ages. In fact there were no more exposures above age 80 for male employees and above age 71 for female employees. So, in order to complete the mortality tables for both the male and female employees, the trend of Hunter's Tropical and the Commissioners' Standard Ordinary mortality tables at the older ages was followed and the rates of mortality for the older ages up to the end of the table were read off from the graphs. Table VI gives the two graduated rates of mortality for the male and female employees based on the experience of the Government Service Insurance System in the observation period July 1, 1951 to July 1, 1954.

Grateful acknowledgment is hereby made to Miss Tala de la Paz, Mr. Adriano Batara and all the employees of the Actuarial Staff of the Government Service Insurance System who did most of the tremendous computational work needed in this investigation.

LIST OF REFERENCES

1. Actuarial Statistics, Vol. I, Statistics and Graduation by H. Tetley, pp. 115 - 152.
2. Mathematical Theory of Graduation by Robert Henderson, pp. 1 - 19.
3. Tables for Statisticians by Arkin and Colton pp. 114, 121.
4. Outline of Valuation Procedures for Retirement Insurance Plan of the Philippine Government Insurance System prepared in 1952 by Prof. C. J. Nesbitt of the University of Michigan, Actuarial Consultant of the System.

GRAPH USED IN GRADUATING MALE RATES OF MORTALITY

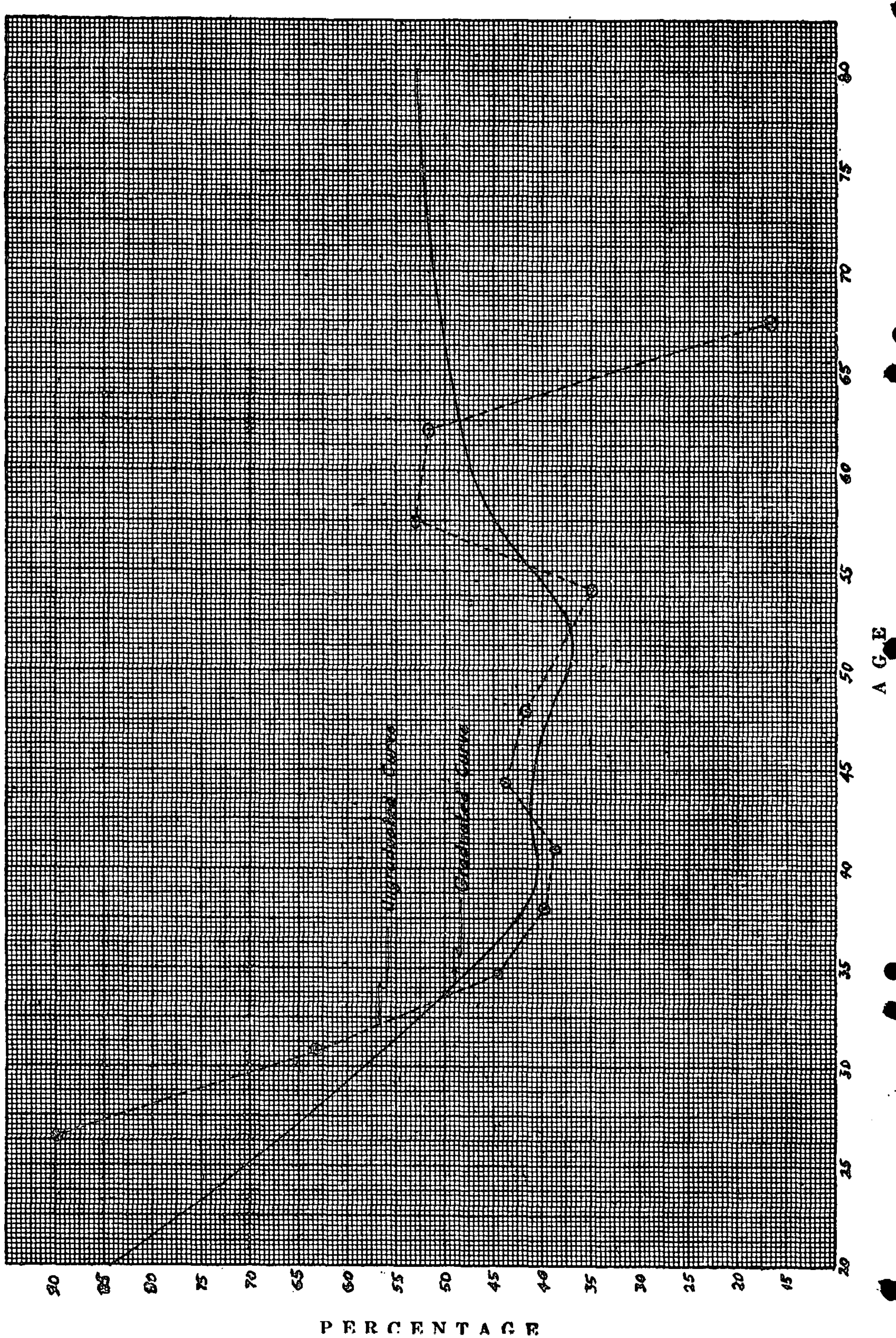

TABLE I

COMPARISON OF ACTUAL TO EXPECTED DEATHS

MALE

Age	E_x	q_x CSO	Expected Deaths	Actual Deaths	Actual: Expected
21	1	.00251	.002	0	0
22	41	.00259	.106	0	0
23	371	.00268	.994	0	0
24	1120	.00277	3.102	2	.645
25	2168	.00288	6.244	6	.961
26	2945	.00299	8.806	11	1.249
27	3339	.00311	10.384	9	.867
28	3541	.00325	11.508	9	.782
29	3774	.00340	12.832	8	.623
30	4117	.00356	14.656	9	.614
31	4532	.00373	16.904	11	.651
32	5265	.00392	20.521	13	.633
33	6083	.00412	25.062	11	.439
34	7219	.00435	31.403	10	.318
35	8044	.00459	36.922	19	.515
36	8115	.00486	39.439	19	.482
37	7688	.00515	39.593	9	.227
38	7141	.00546	38.990	17	.436
39	7326	.00581	42.564	22	.517
40	7425	.00618	45.886	11	.240
41	7741	.00659	51.013	22	.431
42	7588	.00703	53.345	25	.469
43	7415	.00751	55.687	26	.467
44	7428	.00804	59.721	30	.502
45	5537	.00861	47.674	21	.440
46	5070	.00923	46.796	15	.321
47	5102	.00991	50.561	23	.455

Table I — Continued

Age	E_x	q_x CSO	Expected Deaths	Actual Deaths	Actual: Expected
48	5085	.01064	54.104	20	.370
49	4727	.01145	54.124	23	.425
50	3547	.01232	43.699	19	.435
51	3329	.01327	44.176	16	.362
52	3368	.01430	48.162	17	.352
53	3334	.01543	51.444	18	.350
54	3107	.01665	51.732	16	.309
55	2521	.01798	45.328	18	.397
56	2338	.01943	45.427	16	.352
57	2361	.02100	49.581	25	.504
58	2099	.02271	47.668	31	.650
59	2013	.02457	49.459	30	.607
60	1338	.02659	35.577	14	.394
61	1046	.02878	30.104	24	.797
62	925	.03118	28.842	19	.659
63	891	.03376	30.080	12	.402
64	838	.03658	30.654	11	.359
65	535	.03964	21.207	4	.189
66	345	.04296	14.821	1	.067
67	299	.04656	13.921	1	.072
68	252	.05046	12.716	4	.315
69	307	.05470	16.793	4	.238
70	78	.05930	4.625	1	.216
71	20	.06427	1.285	0	0
72	18	.06966	1.254	0	0
73	14	.07550	1.057	0	0
74	11	.08181	.900	0	0
75	6	.08864	.532	0	0
76	2	.09602	.192	0	0
77	3	.10399	.312	0	.0
78	1	.11259	.113	0	0
79	1	.12186	.122	0	0
80	1	.13185	.132	0	0

TABLE II

GROUPING OF EXPOSURES FOR GRAPHIC METHOD

MALE

Age Group	Average Age	Expected Deaths (CSO)	Actual Deaths	Actual: Expected
21 — 28	26.4	41.15	37	89.9%
29 — 32	30.7	64.91	41	63.2
33 — 36	34.7	132.83	59	44.4
37 — 39	38.0	121.15	48	39.6
40 — 42	41.0	150.24	58	38.6
43 — 46	44.4	209.88	92	43.8
47 — 49	48.0	158.79	66	41.6
50 — 52	51.0	136.04	52	38.2
53 — 55	54.0	148.50	52	35.0
56 — 59	57.5	192.14	102	53.1
60 — 64	61.9	155.26	80	51.5
65 — 80	67.5	90.38	15	16.6

TABLE III

GRADUATED DATA

MALE

Age	q_x CSO	Graduated Percentage	Graduated q_x	E_x	Expected Deaths	Actual Deaths	Actual — Expected
21	.00251	78.9%	.00198	1	.002	—	— .002
22	.00259	76.8	.00199	41	.082	—	— .082
23	.00268	74.6	.00200	371	.742	—	— .742
24	.00277	72.6	.00201	1120	2.251	2	— .251
25	.00288	70.1	.00202	2168	4.379	6	1.621
26	.00299	67.8	.00203	2945	5.978	11	5.022
27	.00311	65.5	.00204	3339	6.812	9	2.188
28	.00325	63.0	.00205	3541	7.259	9	1.741
29	.00340	60.8	.00207	3774	7.812	8	.188
30	.00356	58.6	.00209	4117	8.605	9	.395
31	.00373	56.2	.00210	4532	9.517	11	1.483
32	.00392	54.0	.00212	5265	11.162	13	1.838
33	.00412	51.9	.00214	6083	13.018	11	— 2.018
34	.00435	48.4	.00216	7219	15.593	10	— 5.593
35	.00459	47.5	.00218	8044	17.535	19	1.465
36	.00486	45.5	.00221	8115	17.934	19	1.066
37	.00515	43.4	.00224	7688	17.221	9	— 8.221

Table III — Continued

Age	q_x CSO	Graduated Percentage	Graduated q_x	E_x	Expected Deaths	Actual Deaths	Actual — Expected
38	.00546	41.8	.00228	7141	16.281	17	.719
39	.00581	40.8	.00237	7326	17.363	22	4.637
40	.00618	40.6	.00251	7425	18.637	11	—7.637
41	.00659	40.7	.00268	7741	20.746	22	1.254
42	.00703	40.9	.00288	7588	21.853	25	3.147
43	.00751	40.8	.00306	7415	22.996	26	3.004
44	.00804	40.7	.00327	7428	24.290	30	5.710
45	.00861	40.5	.00349	5537	19.324	21	1.676
46	.00923	40.2	.00371	5070	18.810	15	—3.810
47	.00991	39.8	.00394	5102	20.102	23	2.898
48	.01064	39.7	.00416	5085	21.154	20	—1.154
49	.01145	38.2	.00437	4787	20.919	23	2.081
50	.01232	37.3	.00460	3547	16.316	19	2.684
51	.01327	36.7	.00487	3329	16.212	16	— .212
52	.01430	37.0	.00529	3368	17.817	17	— .817
53	.01543	37.9	.00585	3334	19.504	18	—1.504
54	.01665	39.5	.00658	3107	20.444	16	—4.444
55	.01798	41.0	.00737	2521	18.580	18	— .580
56	.01943	42.7	.00830	2338	19.405	16	—3.405
57	.02100	44.3	.00930	2361	21.957	25	3.043
58	.02271	45.5	.01031	2092	21.569	31	9.431
59	.02457	46.4	.01140	2013	22.948	30	7.052
60	.02659	47.0	.01250	1338	16.725	14	—2.725
61	.02878	47.6	.01370	1046	14.330	24	9.670
62	.03118	48.1	.01500	925	13.875	19	5.125
63	.03376	48.5	.01637	891	14.586	12	—2.586
64	.03658	48.9	.01789	838	14.992	11	—3.992
65	.03964	49.4	.01958	535	10.475	4	—6.475
66	.04296	49.7	.02135	345	7.366	1	—6.366
67	.04656	50.0	.02328	299	6.961	1	—5.961
68	.05046	50.4	.02543	252	6.408	4	—2.408
69	.05470	50.7	.02773	307	8.513	4	—4.513
70	.05930	50.9	.03018	78	2.354	1	—1.354
71	.06427	51.2	.03291	20	.658	—	— .658
72	.06966	51.5	.03587	18	.646	—	— .646
73	.07550	51.7	.03903	14	.546	—	— .546
74	.08181	51.9	.04246	11	.467	—	— .467
75	.08864	52.2	.04627	6	.278	—	— .278
76	.09602	52.4	.05031	2	.101	—	— .101
77	.10399	52.5	.05459	3	.164	—	— .164
78	.11259	52.7	.05933	1	.059	—	— .059
79	.12186	53.3	.06495	1	.065	—	— .065
80	.13185	54.2	.07146	1	.071	—	— .071
		TOTALS			702.769	702	—79.907 +79.138

TABLE IV

TEST FOR SMOOTHNESS

Age	$10^5 q_x$	Δ	Δ^2	Δ^3	Age	$10^5 q_x$	Δ	Δ^2	Δ^3
21	198				51	487	27	4	2
22	199	1			52	529	42	15	11
23	200	1	0		53	585	56	14	— 1
24	201	1	0	0	54	658	73	17	3
25	202	1	0	0	55	737	79	6	—11
26	203	1	0	0	56	830	93	14	8
27	204	1	0	0	57	930	100	7	— 7
28	205	1	0	0	58	1031	101	1	— 6
29	207	2	1	1	59	1140	109	8	7
30	209	2	0	— 1	60	1250	110	1	— 7
31	210	1	— 1	— 1	61	1370	120	10	9
32	212	2	1	2	62	1500	130	10	0
33	214	2	0	— 1	63	1637	137	7	— 3
34	216	2	0	0	64	1789	152	15	8
35	218	2	0	0	65	1958	169	17	21
36	221	3	1	1	66	2135	177	8	— 9
37	224	3	0	— 1	67	2328	193	16	8
38	228	4	1	1	68	2543	215	22	6
39	237	9	5	4	69	2773	230	15	— 7
40	251	14	5	0	70	3018	245	15	0
41	268	17	3	— 2	71	3291	273	28	13
42	288	20	3	0	72	3587	296	23	— 5
43	306	18	— 2	— 5	73	3903	316	20	— 3
44	327	21	3	5	74	4246	343	27	7
45	349	22	1	— 2	75	4627	381	38	11
46	371	22	0	— 1	76	5031	404	23	—15
47	394	23	1	1	77	5459	428	24	1
48	416	22	— 1	— 2	78	5933	474	46	22
49	437	21	— 1	0	79	6459	526	52	6
50	460	23	2	3	80	7001	542	16	—36

Summation absolute value of third differences = 268

Average absolute value of third differences $= \dfrac{268}{57} = .000047$

TABLE V

CHI SQUARE TEST

Age	$E_xq_xp_x$	$(A-E)^2$	Chi Square	Age	$E_xq_xp_x$	$(A-E)^2$	Chi Square
21	.002	.000	.000	51	16.133	.045	.003
22	.082	.007	.085	52	17.723	.667	.038
23	.741	.551	.744	53	19.390	2.262	.117
24	2.246	.063	.028	54	20.309	19.794	.972
25	4.370	2.628	.601	55	18.443	.336	.018
26	5.966	25.220	4.227	56	19.244	11.594	.602
27	6.798	4.787	.704	57	21.753	9.260	.426
28	7.244	3.031	.418	58	21.347	88.944	4.167
29	7.796	.035	.005	59	22.683	49.731	2.192
30	8.587	.156	.018	60	16.516	7.426	.450
31	9.497	2.199	.232	61	14.134	93.509	6.616
32	11.138	3.378	.303	62	13.667	26.266	1.922
33	12.990	4.072	.313	63	14.347	6.687	.466
34	15.559	31.282	2.011	64	14.724	15.936	1.082
35	17.497	2.146	.123	65	10.270	41.926	4.082
36	17.894	1.136	.063	66	7.209	40.526	5.622
37	17.182	67.585	3.933	67	6.799	35.534	5.226
38	16.244	.517	.032	68	6.245	5.798	.928
39	17.322	21.502	1.241	69	8.277	20.367	2.461
40	18.590	58.324	3.137	70	2.283	1.833	.803
41	20.690	1.573	.076	71	.636	.433	.680
42	21.790	9.904	.455	72	.623	.417	.670
43	22.926	9.024	.394	73	.525	.298	.569
44	24.211	32.604	1.353	74	.447	.218	.488
45	19.257	2.809	.146	75	.265	.077	.291
46	18.739	14.516	.775	76	.096	.010	.106
47	20.023	8.398	.419	77	.155	.027	.173
48	21.066	1.332	.063	78	.056	.003	.063
49	20.828	4.331	.208	79	.061	.004	.069
50	16.241	7.204	.444	80	.065	.005	.075
							63.928

TEST:

$$\sqrt{2(\text{Chi}^2)} - \sqrt{2n-1}$$

$$= \sqrt{2(63.928)} - \sqrt{2(60)-1}$$

$$= \sqrt{127.856} - \sqrt{119}$$

$$= 11.31 - 10.91 = .40 < 1.7$$

Probability = 34.5%

TABLE VI

GRADUATED RATES OF MORTALITY FOR MALE AND FEMALE

Age	Male	Female	Age	Male	Female
21	.00198	——	64	.01789	.01112
22	.00199	.00075	65	.01958	.01257
23	.00200	.00076	66	.02135	.01448
24	.00201	.00077	67	.02328	.01667
25	.00202	.00078	68	.02543	.01958
26	.00203	.00079	69	.02773	.02308
27	.00204	.00080	70	.03018	.02680
28	.00205	.00082	71	.03291	.03085
29	.00207	.00085	72	.03587	.03201
30	.00209	.00089	73	.03903	.03403
31	.00210	.00093	74	.04246	.03806
32	.00212	.00099	75	.04627	.04209
33	.00214	.00105	76	.05031	.04613
34	.00216	.00112	77	.05459	.05118
35	.00218	.00121	78	.05933	.05624
36	.00221	.00132	79	.06495	.06131
37	.00224	.00144	80	.07146	.06738
38	.00228	.00155	81	.07820	.07246
39	.00237	.00167	82	.08566	.07854
40	.00251	.00179	83	.09386	.08662
41	.00268	.00190	84	.10282	.09370
42	.00288	.00202	85	.11254	.10281
43	.00306	.00213	86	.12302	.11292
44	.00327	.00224	87	.13430	.12403
45	.00349	.00234	88	.14642	.13415
46	.00371	.00244	89	.15942	.14627
47	.00394	.00256	90	.17335	.15840
48	.00416	.00270	91	.18825	.17253
49	.00437	.00288	92	.20418	.18667
50	.00460	.00306	93	.22113	.20281
51	.00487	.00328	94	.23919	.22095
52	.00529	.00350	95	.25846	.23710
53	.00585	.00378	96	.27914	.25725
54	.00658	.00410	97	.30143	.28040
55	.00737	.00444	98	.33183	.30956
56	.00830	.00482	99	.37784	.35072
57	.00930	.00527	100	.44956	.40590
58	.01031	.00577	101	.56699	.48208
59	.01140	.00634	102	.74953	.57727
60	.01250	.00702	103	1.00000	.70046
61	.01370	.00780	104	——	.85066
62	.01500	.00876	105	——	1.00000
63	.01637	.00986			